

Duality and Four-Dimensional Black Holes

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Abstract

We consider the effects of abelian duality transformations on static, spherically-symmetric, asymptotically flat string spacetimes in four dimensions, where the dilaton, axion, metric, and gauge fields are allowed to be nonzero. Independent of the α' expansion, there is a six-parameter family of such configurations, labelled by the charges characterizing the asymptotic behaviour of the various fields: *i.e.* their mass, dilaton charge, axion charge, electric charge, magnetic charge, and Taub-NUT parameter. We show that duality, based on time-translation invariance, maps these solutions amongst themselves, with the effect of interchanging two pairs of these six labels, namely: (1) the mass and dilaton charge, and (2) the axion charge and the Taub-NUT parameter. We consider in detail the special case of the purely Schwarzschild black hole, for which the mass of the dual configuration vanishes to leading order in α' . Working to next-to-leading order in α' for the bosonic and heterotic strings, we find that duality takes a black hole of mass M to a (singular) solution having mass $\sim -1/(\alpha'M)$. Finally, we argue that two solutions which are related by duality based on a noncompact symmetry are *not* always physically equivalent.

1. Introduction

One of the remarkable features of string theory is the surfeit of symmetry that it exhibits. This abundance of symmetry is also reflected in the low-energy equations which govern the so-called massless modes of the string: the dilaton, graviton, axion and gauge fields. Among the most ‘stringy’ of these symmetries are target space duality transformations [1], which in their simplest form are discrete transformations that relate physically equivalent string configurations. Such transformations are guaranteed to exist whenever a spacetime admits a continuous isometry [2].

Duality symmetries are particularly interesting because they have been found to relate string solutions that one would otherwise have thought to be completely unrelated. The original such example is the duality between string propagation on very large and very small toroidal spacetimes [3], but there are many others. Target space duality relates strongly-curved, singular regions and ordinary nonsingular parts of spacetime, such as arise in two-dimensional black holes [4], [5], and in black string spacetimes [6], [7]. Other duality symmetries include mirror symmetry [8], and the conjectured duality between weak and strong coupling in string theory [9].

To date, comparatively little has been done to explore the implications of target space duality for spacetimes in higher — most notably four — dimensions, largely due to lack of knowledge in this case of the explicit form for the solutions to the string equations to all orders in α' , except in certain special cases [10]. (For some recent progress in finding exact solutions see Ref. [11].) Our purpose in the present paper is to partially fill this gap, by examining some of the implications of duality in four (and higher) dimensions. We therefore consider here the generic static, spherically-symmetric, and asymptotically-flat field configurations consisting of the metric, dilaton, axion and electromagnetic fields — in fact below, we relax the restriction that the solutions are static. Such a field configuration always admits a duality transformation that is based on its invariance under time translation (as well as other dualities [12] based on the rotational symmetries), and it is to this duality that we refer in what follows.

We are able to circumvent the obstacle of not knowing the exact solutions to the string equations by working, as much as possible, with the generic field configuration which is consistent with the assumed symmetries, and with asymptotic flatness. We also couch our discussion of the action of duality on these configurations completely in terms of its

implications for the charges of the solutions which describe their asymptotic falloff at spatial infinity. This permits us to describe the exact action of duality in a way which does *not* rely on an explicit knowledge of the functional form of the explicit solutions to all orders in α' . When an explicit form for the fields does become necessary, we use the lowest-order in α' solutions which have been recently found [13] for the six-parameter family of configurations which we consider.

A general time-independent and spherically-symmetric field configuration is completely characterized by a small number of functions of a single (radial) variable, r . Since the string field equations, to lowest order in the α' expansion, are second order partial differential equations, the r dependence of their static solutions is typically determined by the asymptotic behaviour of the various fields as $r \rightarrow \infty$. For asymptotically-flat configurations involving the metric, dilaton, axion and electromagnetic fields, we argue that this asymptotic behaviour is specified by the values of five parameters, or ‘charges’. This counting of parameters is not affected by the inclusion of the higher-order corrections in α' — despite the fact that these typically involve higher derivatives — since we consider only solutions which have a well-behaved limit as α' tends to zero.

The five charges that arise in this way are the configuration’s (i) mass, (ii) dilaton charge, (iii) axion charge, (iv) electric charge and (v) magnetic charge. If we relax the condition that the metric be static, and instead just require it to be stationary,¹ then other parameters become possible. One such is the metric’s Taub-NUT parameter, which we include since, as we shall see, it arises naturally in the action of duality on solutions to the low-energy string equations. This six-parameter family of field configurations includes the string extensions of many familiar spacetimes, such as the Schwarzschild and charged dilatonic black holes. Of these the Schwarzschild black hole is probably the simplest since — to lowest order in α' — all of its charges vanish, except for its mass.

We find that duality has a very simple action on this six-parameter family, interchanging two pairs of the asymptotic constants. In particular, it interchanges (i) the configuration’s mass with its dilaton charge, and (ii) the Taub-NUT parameter with the axion charge. By contrast, the electric and magnetic charges remain invariant. Somewhat surprisingly, we find that these conclusions sensitively depend on the asymptotic behaviour that is assumed for the electromagnetic gauge potential, A_μ , which we take here (in four

¹ We explain this distinction later in the text.

dimensions) to vanish like $1/r$.

Of particular interest is the duality transformation of the nonsingular dilaton–metric black hole of mass M , for which the dilaton charge happens to be zero to leading order in α' . In this case duality produces a solution which is singular at the Schwarzschild radius, and which has a mass which is also zero at this order. Taking into account the $O(\alpha')$ corrections [14], however, we find that the mass M black hole is actually mapped to a solution whose mass is negative, and given by $-k/(\alpha'M)$, with k a positive dimensionless constant for the bosonic and heterotic strings. For the superstring $k = 0$, and so it is necessary to work to higher order in α' in order to determine the mass of the dual solution. We find that this calculation gives a dual mass for the superstring which is again negative, and of order $-1/(\alpha'^3 M^5)$ in size.

This connection, under duality, between solutions with opposite signs for the mass is reminiscent of the duality between the asymptotically flat region (positive mass) and the region beyond the singularity (negative mass) of the 2D black hole, but unlike that case, the 4D black hole seems to map two different geometries instead of two different regions of the same geometry. This might be puzzling if duality transformations were believed to relate physically equivalent configurations, as has been argued for some Wess-Zumino-Witten (WZW) models [15]. Indeed, although the physical equivalence of the dual solutions is expected to hold whenever the symmetry on which the dualization is based is compact [16], their equivalence for noncompact symmetries is not yet clear. We use our explicit dual solutions to argue here that duality could relate physically inequivalent string solutions. One way in which we do so is by demonstrating that they give different results for the scattering, at large radii, of massless string states.

The paper is organized as follows. In the next section we identify the class of field configurations which satisfy our assumed symmetries and boundary conditions. In so doing we identify the six charges which define the asymptotic behaviour of the various fields. We then, in section 3, derive the action of duality on these solutions. Section 4 explores in detail the action of duality for the special case of solutions which involve just the dilaton and metric fields. We start by finding the action of duality on the solutions to the field equations at lowest-order in α' . We then focus on the black-hole solution, *i.e.*, that which contains a nonsingular horizon. Since the mass of the dual solution, in this case, actually vanishes at lowest order in α' , we find the higher-order corrections and obtain the mass of the dual solution to this order. This involves a short digression concerning the $O(\alpha')$ corrections to

the duality transformations themselves. Section 5 addresses the physical (in)equivalence of the dual configurations. We first compute the propagation of massless string states by the general dilaton-metric configuration. We then use the results of this calculation to argue that duality based on noncompact symmetries may not relate physically equivalent string backgrounds. Our conclusions are finally summarized in section 6. We discuss in an appendix the $d > 4$ case for the metric–dilaton–electromagnetic gauge-field system.

2. Static Spherically Symmetric String Solutions

The three fields which appear (in string perturbation theory) in the massless spectrum of a generic string theory consist of the metric, $G_{\mu\nu}$, the dilaton, ϕ , and an antisymmetric Kalb-Ramond field, $B_{\mu\nu}$. In the heterotic string these can also be supplemented by one or more gauge potentials, A_μ . Additional moduli are also possible for specific string vacua. The low-energy action for these fields can be written, in four spacetime dimensions, as [17]:

$$\mathcal{L} = \frac{1}{8\pi\alpha'} \sqrt{-G} e^\phi \left[R(G) + (\nabla\phi)^2 - \frac{1}{12} H^{\mu\nu\lambda} H_{\mu\nu\lambda} - \frac{1}{8} F^{\mu\nu} F_{\mu\nu} \right] + \dots, \quad (1)$$

where $H_{\mu\nu\lambda} = \partial_\mu B_{\nu\lambda} - \frac{1}{4} A_\mu F_{\nu\lambda} +$ (cyclic permutations), and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ are the appropriate field strengths. $R(G)$ here denotes the Ricci scalar for the ‘sigma-model’ metric, $G_{\mu\nu}$, and $\sqrt{-G} = \sqrt{-\det G_{\mu\nu}}$ denotes the usual volume element. The ellipses in this equation represent terms which involve other massless fields and/or terms involving more derivatives that arise at higher orders in the α' expansion. Since no cosmological constant is included here, we imagine that our solutions are complemented by some conformal field theory, such as a toroidal or Calabi–Yau [18] compactification, whose central charge ensures conformal invariance on the world-sheet.

For future use we record here the rescaling that takes one to the ‘Einstein frame’, for which the coefficient of the scalar curvature in the action is independent of the dilaton, ϕ :

$$g_{\mu\nu} \equiv e^\phi G_{\mu\nu}. \quad (2)$$

2.1) Static Spherically Symmetric Configurations

In four spacetime dimensions, the most general static and spherically symmetric metric may always be written [19], [20]

$$dS^2 \equiv G_{\mu\nu} dx^\mu dx^\nu = -F(r) dt^2 + G(r) dr^2 + H(r)(d\theta^2 + \sin^2 \theta d\varphi^2). \quad (3)$$

In fact, the additional freedom in reparameterizing the radial coordinate is commonly used to fix either $H(r) = r^2$ or $H(r) = r^2 G(r)$, but neither choice is particularly convenient for our present analysis. Similarly, the generic time-independent and spherically symmetric configuration for the other fields is

$$\phi = \phi(r), \quad F_{tr} = E(r), \quad F_{\theta\varphi} = B(r) \sin \theta. \quad (4)$$

For the Kalb-Ramond field, the time-independent, spherically symmetric configurations are most easily identified through the scalar ‘axion’ field which is dual to the antisymmetric tensor field:

$$H_{\mu\nu\rho} = -e^{-\phi} \epsilon_{\mu\nu\rho\kappa} \nabla^\kappa a \quad (5)$$

where ϵ is the volume form for the sigma-model metric. Now the generic time-independent, spherically symmetric configuration includes: $a = a(r)$.

Given these configurations, the corresponding Einstein metric is

$$ds^2 \equiv g_{\mu\nu} dx^\mu dx^\nu = -f(r) dt^2 + g(r) dr^2 + h(r)(d\theta^2 + \sin^2 \theta d\varphi^2), \quad (6)$$

where $f = F e^\phi$, $g = G e^\phi$ and $h = H e^\phi$.

There are two important kinds of field configurations which we omit with the above symmetry ansatz. The first of these is the possible topological Kalb-Ramond configuration, $B_{\theta\phi} = Q_{\text{top}} \sin \theta$, for which the corresponding curl vanishes: $dB = 0$. Such a field configuration is not pure gauge provided that the second homotopy group of the background spacetime is nontrivial [21].

The second class of configurations which need not be captured by the above ansatz are those spacetimes which are time-independent, but are not static. That is to say, those for

which it is impossible to choose a time coordinate such that the ‘time-space’ components of the metric vanish everywhere. Such spacetimes are often called stationary.

Since we shall find that duality transformations need not preserve the static form for the metric, we wish to broaden our metric ansatz to include some of these more general configurations. In particular we choose to work with a stationary line element which is reminiscent of the ‘Taub-NUT’ metric [22], [23]:

$$dS^2 \equiv G_{\mu\nu} dx^\mu dx^\nu = -F(r) (dt + 2N \cos \theta d\varphi)^2 + G(r) dr^2 + H(r)(d\theta^2 + \sin^2 \theta d\varphi^2), \quad (7)$$

and similarly for the Einstein metric. The parameter N which appears here we call the Taub-NUT parameter of the metric. Clearly, the static case corresponds to the choice $N = 0$. The metric of eq. (7) is still invariant under time translations and $SO(3)$ ‘rotations’ [23], but the latter rotations also act on the time coordinate in order to preserve the differential $dt + 2N \cos \theta d\varphi$. Thus spherical symmetry in the conventional sense is lost. Further the time coordinate must be periodically identified with period $8\pi N$ in order to avoid conical singularities at the axes $\theta = 0$ and $\theta = \pi$ [23].

2.2) Asymptotic Forms

The requirement of asymptotic flatness imposes some conditions on the limiting forms as $r \rightarrow \infty$ of the various functions that appear in the above *ansätze* for the low energy fields. In particular, in this limit we may choose the metric functions F and G to approach unity, and the function H to approach r^2 . The remaining fields are also restricted since asymptotic flatness requires the stress-energy tensor to approach zero at large radii.

Therefore we take the two scalar fields, ϕ and a , to approach constants for large r . Since the axion field, a , as defined by eq. (5), appears in equations of motion only through its derivative, the theory admits the symmetry $a \rightarrow a + c$, for constant c . It is convenient to use this symmetry to ensure that a approaches zero as $r \rightarrow \infty$.

Similarly, it is also convenient to absorb the asymptotic value, ϕ_0 , of the dilaton field into the definition for Newton’s constant, G_N . For instance, comparing the action of eq. (1) with the standard Einstein-Hilbert form gives: $G_N = \frac{1}{2} e^{-\phi_0} \alpha'$. With this choice we may also take the dilaton field to vanish at infinity, in which case the large- r limit of the functions f and g of the Einstein metric is also unity.

Finally, we require the electric and magnetic fields to fall to zero at infinity, and this is accomplished if the component F_{tr} of the electromagnetic field strength falls to zero, while the component $F_{\theta\varphi}$ approaches $Q_M \sin \theta$, for Q_M a constant.

Solving the low-energy string equations in four spacetime dimensions and imposing these asymptotic flatness conditions, typically produces fields which approach their asymptotic values like $1/r$. Therefore the solutions exhibit the following asymptotic behaviour:

$$F(r) = 1 - \frac{A}{r} + \dots; \quad G(r) = 1 + \frac{B}{r} + \dots; \quad e^{\phi(r)} = 1 - \frac{Q_D}{r} + \dots, \quad (8)$$

and $H(r) = r^2 [1 + O(\frac{1}{r})]$.

The above relations imply similar ones for the Einstein metric:

$$f(r) = 1 - \frac{\mathcal{A}}{r} + \dots; \quad g(r) = 1 + \frac{\mathcal{B}}{r} + \dots; \quad h(r) = r^2 \left[1 + O\left(\frac{1}{r}\right) \right], \quad (9)$$

with $\mathcal{A} = A + Q_D$ and $\mathcal{B} = B - Q_D$.

The asymptotic form for the remaining fields is taken to be

$$a(r) = -\frac{Q_A}{r} + \dots; \quad F_{tr} = \frac{Q_E}{r^2} + \dots, \quad \text{and} \quad F_{\theta\varphi} = Q_M \sin \theta + \dots. \quad (10)$$

The ellipsis in the above expressions indicate terms of higher order in $1/r$. The constants Q_A , Q_E and Q_M represent the configuration's total axion, electric and magnetic charges, respectively. Similarly in eq. (8), Q_D is called the dilaton charge. Since the duality transformations in the presence of electromagnetic fields are given in terms of the gauge potential, A_μ , it is sometimes necessary to work directly with the asymptotic form for this field. Explicitly, we take

$$A_t = \frac{Q_E}{r} + \dots, \quad \text{and} \quad A_\varphi = -Q_M \cos \theta + \dots, \quad (11)$$

while the remaining components vanish. In particular notice that we take A_t to vanish as $r \rightarrow \infty$. As we shall see, many results turn out to be sensitive to a nonzero value for A_μ at large r . We also require the asymptotic form of the antisymmetric tensor field corresponding to $a(r)$ in eq. (10),

$$H_{t\theta\varphi} = -Q_A \sin \theta + \dots; \quad \text{and} \quad B_{\varphi t} = Q_A \cos \theta + \dots, \quad (12)$$

while again the remaining components vanish.

The asymptotic forms for the metric have a physical significance in that they determine the mass of the corresponding configuration. The spatial components of the Einstein metric determine the conserved (ADM) energy [24], or ‘inertial mass’, M_I , according to the formula

$$M_I = \frac{\mathcal{B}}{2G_N}. \quad (13)$$

This is the mass which emerges in a calculation of the energy using a gravitational stress-energy pseudo-tensor, [20]. The asymptotic form of g_{tt} also has a significance, in that it determines the ‘gravitational mass’, M_G , which controls the Newtonian limit of the motion of nonrelativistic particles along geodesics in the asymptotically-flat region. It is given by

$$M_G = \frac{\mathcal{A}}{2G_N} . \quad (14)$$

Notice we use the asymptotic form for the Einstein metric, $g_{\mu\nu}$, in the above expressions. Since we know that as $\alpha' \rightarrow 0$ the equations of motion for $g_{\mu\nu}$ reduce to Einstein’s equations with a conventional stress-energy tensor, we are guaranteed that $M_I = M_G$ for the string solutions. The latter result would not be true in general if the same definitions were applied to the sigma-model metric. In that case, terms linear in the dilaton enter stress-energy tensor, and can lead to a difference between M_I and M_G [14].

Finally, we remind the reader that the Taub-NUT parameter is determined by the asymptotic behavior: $G_{t\varphi} = -2N \cos \theta + \dots$.

3. Duality

We next turn to the behaviour of these solutions under duality transformations based on time-translation symmetry. In order to do so we first describe the transformation rules themselves.

3.1) The Transformation Rules

The existence of a continuous symmetry in a solution to the low-energy string equations immediately permits the construction of another, dual, solution to the same equations

[1]. If the symmetry on which this duality construction is based should be compact, then the two configurations that are related in this way represent precisely the same conformal field theory, and so are two representations of a single string background [16]. If the symmetry is noncompact, however, then the physical equivalence of the dual solutions is not so clear. One of our motivations for studying duality for higher-dimensional black holes is to pursue this issue in a concrete setting.

If all of the fields are independent of the time coordinate, the action of the duality transformation for a nontrivial configuration involving the metric, dilaton and antisymmetric tensors is given by [2]:

$$\begin{aligned}\tilde{G}_{tt} &= 1/G_{tt}, & \tilde{G}_{ti} &= -B_{ti}/G_{tt}, & \tilde{G}_{ij} &= G_{ij} - \frac{G_{ti}G_{tj} - B_{ti}B_{tj}}{G_{tt}} \\ \tilde{B}_{ti} &= -G_{ti}/G_{tt}, & \tilde{B}_{ij} &= B_{ij} + \frac{G_{ti}B_{tj} - G_{tj}B_{ti}}{G_{tt}}, & e^{\tilde{\phi}} &= e^{\phi} \left(\frac{\det G}{\det \tilde{G}} \right)^{1/2}\end{aligned}\quad (15)$$

where ‘ t ’ denotes the time direction.² Ref. [27] presents these transformations in a manifestly covariant framework based on the Killing vector associated with the time-translation symmetry.

The duality transformation is more complicated than eqs. (15) when gauge fields, A_μ , are present, however. In this case, eqs. (15) may still be used, but with the proviso that $G_{\mu\nu}$ is to be replaced everywhere by the quantity $G_{\mu\nu} + \frac{1}{4}A_\mu A_\nu$. The only exception to this substitution is in the dilaton transformation, which remains exactly the same as in eq. (15). The gauge potentials in the dual solution are given by [28]:

$$\begin{aligned}\tilde{A}_t &= -\frac{A_t}{G_{tt} + \frac{1}{4}A_t A_t} \\ \tilde{A}_i &= A_i - A_t \frac{G_{ti} + \frac{1}{4}A_t A_i - B_{ti}}{G_{tt} + \frac{1}{4}A_t A_t}.\end{aligned}\quad (16)$$

Two uncertainties hang over the above transformation rules in the presence of gauge fields. Firstly, reference [28] considers only toroidal compactifications, and so a derivation

² Our transformation of the ‘ $t-i$ ’ components of the fields differs from that of Ref. [2] by a sign. They are nevertheless equivalent since this sign may be compensated by performing the coordinate transformation $t \rightarrow -t$ in the dual solution. A similar result was found in Refs. [25] and [26]. Note that if we had made the alternate choice for these signs, then the sign of the electric charge would be reversed under a duality transformation.

of these transformation rules for gauge fields in the general case, along the lines of Ref. [2], is still lacking. One way to convince oneself that Ref. [28]’s transformation rules are nonetheless correct in the general case, however, is to identify the duality transformation as a discrete automorphism of a larger class of $O(d, d+p)$ symmetries of the heterotic string, as is done in Ref. [29]. An independent construction of these gauge field transformations begins with a Kaluza-Klein compactification of the $(D + 1)$ -dimensional theory, and then makes a consistent truncation (see for example the first article in Ref. [11]) leaving the D -dimensional theory with one extra $U(1)$ gauge field. The above transformations (16) then follow directly from the standard duality rules (15) of the higher dimensional theory³.

The second potential difficulty arises because other workers [30] have arrived at a different set of transformation rules in the presence of gauge fields (following Ref. [2]). We believe the origin of the nominal contradiction between the results of Ref. [30] on one hand, and Refs. [28] and [29] on the other lies in their different treatment of the size of the gauge potential in the α' expansion. In one approach, gauge fields contribute at higher orders in the α' expansion of the low energy string equations. This feature is important in determining the supersymmetry of solutions, such as for Calabi-Yau compactifications. Thus in Ref. [30], where the focus is on supersymmetry, it appears that background gauge fields produce α' corrections to the leading order duality transformations given in eqs. (15). In some circumstances, such as when studying black hole configurations, one considers very large electromagnetic charges of $O(1/\sqrt{\alpha'})$, which then contribute in the leading order equations. In this case, it is convenient to rescale A_μ so that the gauge fields appear amongst the field equations at leading order in α' . In the latter approach, we believe the results of Refs. [28] and [29] apply.

3.2) Asymptotic Forms in Four Dimensions

The above duality transformations have the property that they preserve the large- r behaviour of the fields we are considering. In particular, the form of the line element given in eq. (7) is preserved. So duality takes any configuration from the family of solutions whose asymptotic forms are labelled by the parameters, A , B , (or \mathcal{A} and \mathcal{B} , for the Einstein metric), Q_D , Q_A , Q_E , Q_M and N , onto another field configuration that is also

³ We thank A. Tseytlin for this observation.

labelled by (usually different) values of the same parameters. Hence the effect of the duality transformation is conveniently expressed through its action on the physical charges characterizing the asymptotic fields.

If the parameters which define the dual solution are denoted by \tilde{A} , \tilde{B} (or $\tilde{\mathcal{A}}$ and $\tilde{\mathcal{B}}$ for the Einstein metric) \tilde{Q}_D , \tilde{Q}_A , \tilde{Q}_E , \tilde{Q}_M and \tilde{N} , then eqs. (15) and (16) imply these are given by:

$$\tilde{A} = -A, \quad \tilde{B} = B, \quad \tilde{Q}_D = Q_D + A, \quad \tilde{Q}_A = 2N, \quad 2\tilde{N} = Q_A, \quad (17)$$

while $\tilde{Q}_E = Q_E$ and $\tilde{Q}_M = Q_M$.

In terms of the constants \mathcal{A} and \mathcal{B} which give the asymptotic behaviour of the dual Einstein metric the above equations become

$$\tilde{\mathcal{A}} = Q_D, \quad \tilde{\mathcal{B}} = \mathcal{B} - \mathcal{A} + Q_D, \quad \tilde{Q}_D = \mathcal{A}. \quad (18)$$

These last relations can be re-expressed, using eqs. (13) and (14), in terms of the masses and dilaton charge. We then see quite generally that, for an *arbitrary* member of our six-parameter family of field configurations, duality has the following action upon the asymptotic charges:

$$2G_N M \leftrightarrow Q_D, \quad Q_A \leftrightarrow 2N, \quad (19)$$

with Q_E and Q_M fixed. That is, duality interchanges the mass with the dilaton charge, as well as interchanging the axion charge with the Taub-NUT parameter.

We stress that since this result relies just on the asymptotic form for the fields involved, it is completely general and applies equally well to exact string solutions as to lowest-order ones. One surprise is that the relations we have derived amongst the various charges depend on the asymptotic value of the electrostatic potential, A_t — we discuss this in more detail below. Our present results assume that A_t vanishes like $1/r$ for large r . As we shall see later, if this large- r behaviour is relaxed — with A_t approaching a nonzero constant for large r — then, for example, Q_E need not be invariant under duality. (Duality transformations for which Q_E is not invariant have been considered for two-dimensional configurations in Refs. [31] and [1].)

A nice illustration of these general relations that has been considered in the literature is the case of the magnetically charged dilatonic black hole in four dimensions [32], for which the dilaton charge, Q_D , turns out to be related to the mass, M , and the magnetic charge, Q_M , by $Q_D = Q_M^2/(G_N M)$. Here (assuming a gauge for which $A_t = 0$) the dual solution is obtained from the original one by everywhere making the replacement [12]:

$$2G_N M \rightarrow 2G_N \widetilde{M} = \frac{Q_M^2}{2G_N M}, \quad \text{and} \quad \frac{Q_M^2}{2G_N M} \rightarrow \frac{\widetilde{Q}_M^2}{2G_N \widetilde{M}} = 2G_N M. \quad (20)$$

Notice that eqs. (20) — together with the expression for Q_D in terms of Q_M and $G_N M$ — is a special case of the general transformation rules derived here: $(2G_N M, Q_D, Q_M) \rightarrow (2G_N \widetilde{M}, \widetilde{Q}_D, \widetilde{Q}_M) = (Q_D, 2G_N M, Q_M)$. In this particular example, the naked singularity domain, $Q_M^2 > 2G_N^2 M^2$, is mapped to the black hole domain, $Q_M^2 < 2G_N^2 M^2$, similar to what happens in two-dimensional examples. The extreme case $Q_M^2 = 2G_N^2 M^2$ is a selfdual solution.

4. The Dilaton–Metric Case in Detail

It is instructive to work out the previous general manipulations in a concrete example. We do so here using the explicit solution for the dilaton–metric system to lowest-order in α' . The results we obtain lead us to consider in some detail the dualization of the Schwarzschild black hole, including its nonleading α' corrections.

4.1) Duality and the Lowest-Order Four-Dimensional Solutions

We record here, for future use, the dilaton–metric string solutions to the lowest-order in α' string equations in four dimensions. (The higher-dimensional case is treated in the appendix.) The relevant field equations arise from varying the low-energy action of eq. (1), and are given by

$$R_{\mu\nu}(g) = \frac{1}{2} \nabla_\mu \phi \nabla_\nu \phi \quad (21)$$

$$\nabla^2 \phi = 0.$$

The static, spherically-symmetric and asymptotically-flat solutions to these equations

are [33], [13] given by the ansatz of eq. (6) with:

$$\begin{aligned} f = \frac{1}{g} &= \left(1 - \frac{\ell}{r}\right)^\delta \\ h &= r^2 \left(1 - \frac{\ell}{r}\right)^{1-\delta} \\ e^\phi &= \left(1 - \frac{\ell}{r}\right)^\gamma \end{aligned} \tag{22}$$

where ℓ , δ and γ are arbitrary constants, apart from the one condition $\delta^2 + \gamma^2 = 1$. The mass and dilaton charge of these solutions are given by $2G_N M = \delta\ell$ and $Q_D = \gamma\ell$. For $\ell > 0$ the choice $(\delta, \gamma) = (1, 0)$ corresponds to the positive-mass Schwarzschild solution, and $(\delta, \gamma) = (-1, 0)$ also yields the Schwarzschild solution up to a coordinate transformation, but with a negative mass.

Notice that when $\gamma \neq 0$, and so $Q_D \neq 0$, these field configurations contain a curvature singularity at $r = \ell$, as can be seen from the equations of motion:

$$\begin{aligned} R(g) &= \frac{1}{2} (\nabla\phi)^2 \\ &= \frac{\gamma^2 \ell^2}{2r^4} \left(1 - \frac{\ell}{r}\right)^{\delta-2}. \end{aligned} \tag{23}$$

We now ask how these solutions transform under duality. Applying the rules of the previous section, the dual turns out to be given by

$$\begin{aligned} \tilde{f} = \frac{1}{\tilde{g}} &= \left(1 - \frac{\ell}{r}\right)^\gamma \\ \tilde{h} &= r^2 \left(1 - \frac{\ell}{r}\right)^{1-\gamma} \\ e^{\tilde{\phi}} &= \left(1 - \frac{\ell}{r}\right)^\delta \end{aligned} \tag{24}$$

Thus the duality transformation simply interchanges δ and γ . This is just what is required to ensure that the mass and dilaton charge are exchanged according to the general result $2G_N M \leftrightarrow Q_D$. The special cases $\delta = \gamma = \pm \frac{1}{\sqrt{2}}$ are distinguished by being self dual.

The previous discussion shows that the only solutions for which $r = \ell$ is nonsingular are those for which the dilaton charge, Q_D , vanishes. The positive-mass example of this

is the solution for which $(\delta, \gamma) = (1, 0)$ — *i.e.* the positive-mass Schwarzschild black hole. The dual solution, in this case, has $(\tilde{\delta}, \tilde{\gamma}) = (0, 1)$, which is clearly one of the configurations having a curvature singularity at $r = \ell$.

The result $\tilde{\delta} = 0$ implies, in particular, that the mass, \widetilde{M} , of the dual solution vanishes, at least to lowest order in α' . In order to gain some intuition for the physics of this dual configuration, we now compute the dual mass to next order in α' . We find the intriguing result that the dual mass is given in terms of the original mass, M , by the relation

$$\widetilde{M} = -\frac{k}{\alpha' M}, \quad (25)$$

where k is a known non-negative number. It is the purpose of the remainder of this section to prove this result, and to determine k . Before exploring the $O(\alpha')$ corrections to the lowest-order string solutions, however, it is first necessary to make a short aside concerning the necessity of also modifying the duality transformation rules at $O(\alpha')$, which one might expect to modify the physical charges at this order.

4.2) $O(\alpha')$ Corrections to the Duality Transformations

The leading corrections to the effective action that are relevant for modifying nontrivial gravitational solutions are [17]:

$$I = \int d^4x \sqrt{-G} e^\phi \left[R(G) + (\nabla\phi)^2 + \frac{\lambda}{2} \alpha' R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \dots \right], \quad (26)$$

where λ is, respectively, $\frac{1}{2}$ for bosonic, $\frac{1}{4}$ for heterotic, and 0 for supersymmetric strings. There are also higher-derivative terms that enter the effective action at this order through the Lorentz Chern-Simons contributions to $H_{\mu\nu\rho}$, and through field redefinition ambiguities. None of these terms are relevant for the present discussion, which is restricted to static, uncharged, dilaton–metric configurations in four dimensions.

Although the duality transformation given by eqs. (15) is a symmetry of the leading order terms in the action, it is *not* a symmetry of the $O(\alpha')$ correction for general curved backgrounds. The variation of the action turns out to be proportional to the field equations themselves, however, indicating that the action can be made invariant by modifying the duality transformation rules at $O(\alpha')$ [34].

For a sigma-model metric of the form,

$$dS^2 = -F(r) dt^2 + G(r) dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (27)$$

the corrected duality transformation (associated with time translations) which produces a symmetry of the action, including the $O(\alpha')$ interactions is:

$$\tilde{F}(r) = \frac{1}{F(r)} \left(1 + \frac{\lambda\alpha'}{GF^2} \left(\frac{dF}{dr} \right)^2 \right), \quad \tilde{G}(r) = G(r), \quad (28)$$

and

$$\tilde{\phi}(r) = \phi(r) + \ln |F(r)| - \frac{\lambda\alpha'}{2GF^2} \left(\frac{dF}{dr} \right)^2. \quad (29)$$

Of course, these transformations are understood to be applied perturbatively in α' , and one expects that they receive further modifications at $O(\alpha'^2)$ and beyond.

In the present case where $F = 1/G = 1 - \frac{\ell}{r}$, we find

$$\tilde{F} = \frac{1}{1 - \frac{\ell}{r}} \left[1 + \left(\frac{\lambda\alpha'}{1 - \frac{\ell}{r}} \right) \frac{\ell^2}{r^4} \right] \quad (30)$$

and, since $\phi(r) = 0$,

$$\tilde{\phi} = \ln \left(1 - \frac{\ell}{r} \right) - \left(\frac{\lambda\alpha'}{1 - \frac{\ell}{r}} \right) \frac{\ell^2}{2r^4}. \quad (31)$$

Thus we see that the $O(\alpha')$ corrections to the duality transformation only introduce modifications in the asymptotic metric and dilaton at $O(\frac{\alpha'\ell^2}{r^4})$ in eqs. (30) and (31).⁴ As a result, these modifications do not affect the leading asymptotic behavior of the fields, including the gravitational and inertial masses, and the dilaton charge. We expect this to be a general feature of any higher-order corrections to the duality transformation. Any $O(\alpha')$ terms involve two derivatives of the fields, and so such terms cannot affect the

⁴ As was noted in Ref. [34], the $O(\alpha')$ corrections to the duality transformations can be re-expressed so as to preserve the previous duality transformation rule, $\tilde{F}=1/F$, provided that the transformation is combined with an $O(\alpha')$ coordinate change. This extra coordinate transformation also introduces $O(\frac{\alpha'\ell}{r^3})$ modifications into the metric.

leading $O(1/r)$ asymptotic behavior. Hence the physical parameters, such as the mass or dilaton charge, remain unchanged by these corrections. This result should apply in higher dimensions as well.

4.3) The $O(\alpha')$ Corrected Solution

The implications of the $O(\alpha')$ corrections to the string equations arising from eq. (26) for the Schwarzschild black hole are given in Refs. [14]. The main new feature of these corrections is that the dilaton can no longer be held constant, since the (nonvanishing) Riemann tensor provides a source for this field. As a result, the coordinate singularity at the horizon of the lowest-order solution generically becomes a real singularity, unless the asymptotic form of the dilaton field is adjusted to ensure that this does not happen. This tuning of the dilaton corresponds to the freedom to introduce an arbitrary dilaton charge into the solution, and with a particular choice for Q_D at $O(\alpha')$, the horizon singularity is once again purely a problem with coordinates. Refs. [14] give the full form of the resulting solution in any number of dimensions given this choice, but it suffices for our own purposes to present here only the asymptotic form for the result in four dimensions. This is given by eqs. (8), with constants given by:

$$A = \ell + \frac{23\lambda\alpha'}{6\ell} + O(\alpha'^2), \quad B = \ell - \frac{\lambda\alpha'}{6\ell} + O(\alpha'^2), \quad Q_D = -\frac{2\lambda\alpha'}{\ell} + O(\alpha'^2); \quad (32)$$

from which we also learn the form for the Einstein metric, namely eq. (9), with:

$$2G_N M = \mathcal{A} = \mathcal{B} = \ell + \frac{11\lambda\alpha'}{6\ell} + O(\alpha'^2). \quad (33)$$

Recall that the constant λ appearing here is $\frac{1}{2}$ in the bosonic string, $\frac{1}{4}$ in the heterotic string, and 0 in the superstring.

4.4) The Duality Transformation

The action of a duality transformation on the $O(\alpha')$ -corrected solution of the previous section may be obtained as a special case of its action on the asymptotic form for a general configuration. This is given in the present instance by setting $Q_A = Q_E = Q_M = N = 0$ in

eqs. (17), and using expressions (32) and (33) for Q_D and M . This leads to the following expressions for the asymptotic form of the dual solution:

$$\tilde{A} = -\ell - \frac{23\lambda\alpha'}{6\ell} + O(\alpha'^2), \quad \tilde{B} = \ell - \frac{\lambda\alpha'}{6\ell} + O(\alpha'^2), \quad \tilde{Q}_D = \ell + \frac{11\lambda\alpha'}{6\ell} + O(\alpha'^2); \quad (34)$$

and so

$$2G_N \tilde{M} = \tilde{A} = \tilde{B} = -\frac{2\lambda\alpha'}{\ell} + O(\alpha'^2). \quad (35)$$

A comparison of eqs. (33) and (35) gives the advertised relation between the masses of the dual solutions:

$$\tilde{M} = -\frac{k}{\alpha' M} + O\left(\frac{1}{\alpha'^2 M^3}\right), \quad (36)$$

where k is a dimensionless constant, given explicitly by: $k = \lambda\alpha'^2/2G_N^2 = 2\lambda e^{2\phi_0}$. This result is similar to what has been found in two-dimensional examples [5]. The mapping of positive to negative masses under duality was also found for other circumstances in Ref. [25]. Notice, though, that this mapping is *not* a generic property, since it fails for those solutions in eq. (22) for which δ and γ have the same sign.

For the superstring, where $\lambda = 0$, even the $O(\alpha')$ corrections are insufficient to determine the sign of the mass of the dual configuration. For this particular case, the leading corrections to the low energy string equations are $O(\alpha'^3)$ [35], and one must work to this order to determine the mass of the dual solution. The implications of these corrections for black hole solutions have also been determined [36]. With these results in hand and repeating the previous analysis, one finds that

$$\tilde{M} = -3\zeta(3) \frac{\alpha'^3}{(2G_N)^6 M^5} = -3\zeta(3) \frac{e^{6\phi_0}}{\alpha'^3 M^5}, \quad (37)$$

where $\zeta(z)$ denotes the Riemann zeta function. Once again the dual mass is negative.

For the two-dimensional black hole it is also true that the positive-mass black hole is dualized to a negative-mass solution. In the two-dimensional case, however, this relation between positive- and negative-mass solutions is related to the mapping under duality of the asymptotically flat region to the region ‘beyond’ the singularity at $r = 0$ — both of which are incorporated as different sectors of a single conformal field theory. A similar statement for the higher-dimensional case of interest here would be very interesting

but, unfortunately, must await more detailed knowledge concerning the solutions, and the corresponding conformal field theories.

5. The (In)Equivalence of Dual Solutions

An important question for these investigations is whether or not dual solutions describe identical physics. This question is crucial if duality transformations are to be used to infer the properties of string solutions which would otherwise not be amenable to direct analysis. One instance for which there is good evidence for this equivalence is when the orbits of the symmetry on which duality is based are compact [16]. In this case the two dual solutions have been argued to represent exactly the same conformal field theory, and so to represent identical string physics.

Unfortunately, this result cannot be directly applied to the present case, since the time-translation symmetry, $t \rightarrow t + c$, on which our dualization is based, is not compact. In fact, our result that positive-mass solutions get taken to negative-mass ones might make one wonder whether such ‘noncompact’ duality transformations need relate physically equivalent situations at all. The present section is devoted to further exploring this issue within the concrete setting of the dilaton–metric configurations that are discussed above.

We do this exploration in two steps. First, we compute the scattering of massless particles by the four-dimensional dilaton-metric configurations. Part of our purpose for doing so is to verify the physical significance of the result $M_G \rightarrow \widetilde{M}_G = -k/(\alpha' M_G)$, for the gravitational mass, M_G , by rederiving it within the context of a directly observable process. After all, one might wonder whether the existence of the long-range dilaton field might alter particle propagation in such a way as to compensate for the change in the gravitational mass that occurs in passing from a particular dilaton–metric background to its dual. We find that this is not how things work, since the dilaton drops out of the propagation equations for massless particles in the geometric optics approximation. Our second step is to examine several arguments in favour of the inequivalence of such solutions.

5.1) Massless Particle Scattering

Consider, then, using a massless string state as a test particle with which to probe the background metric and dilaton fields. We wish to compute the scattering of such a state

by these field configurations, with the goal of comparing the result for backgrounds that are related by duality.

The first question we must answer concerns how the background fields affect the motion of such test particles. The classical propagation of a closed string through a given background is determined by finding the string world-sheet which is a stationary point of the Polyakov string action, and which has the topology of a cylinder. This world sheet action depends on the background dilaton field configuration, $\phi(x)$, through the coupling term [17]:

$$-\frac{1}{8\pi} \int d^2\sigma \sqrt{-h} \phi[x(\sigma)] \mathcal{R}, \quad (38)$$

where $x^\mu(\sigma)$ are the coordinates of the string world sheet, $h = \det h_{ab}$ is the determinant of the world-sheet metric, and \mathcal{R} is its intrinsic curvature. But the cylindrical worldsheet of a classically propagating string is conformally flat, and so \mathcal{R} can be taken to everywhere vanish. This implies that the dilaton is completely irrelevant for describing such classical string motion.

Furthermore, for a test string for which only massless modes are excited, it is a good approximation to neglect the dependence of $x^\mu(\sigma)$ on the spatial coordinate along the string, and to consider it to be simply a function of the proper time, $\tau = \sigma^0$, of its centre of mass. In this case, the classical equations which determine $x^\mu(\tau)$ reduce to the equations for a null geodesic of the background spacetime metric. Since null geodesics are not affected by conformal rescalings of the metric, it is immaterial whether this null geodesic is computed using the sigma-model metric, $G_{\mu\nu}$, or the Einstein metric, $g_{\mu\nu}$.

An alternative argument to the same end goes as follows. One could instead think of test-string propagation in terms of the four-dimensional effective theory of eq. (1), which describes the low-energy propagation of massless string states. In this case, the classical propagation of, say, a photon is described, to lowest order in α' , by the Maxwell equations $\nabla^\mu (e^\phi F_{\mu\nu}) - \frac{1}{2} \epsilon_{\nu\mu\rho\sigma} (\nabla^\mu a) F^{\rho\sigma} = 0$. For photons with wavelengths that are much longer than the string scale, but also much smaller than the radius of the background curvature or the scale of variation of the background dilaton and axion fields, these equations may be solved in the geometrical-optics approximation [20][37]. This leads, once again, to the null geodesics which are the same for the sigma-model and Einstein metrics.

Using either line of argument, it is clear that the background dilaton field cannot

affect the scattering of massless test strings in the regime we are considering. Consider, therefore the scattering of such a massless string state using the sigma-model metric of eq. (3):

$$dS^2 \equiv G_{\mu\nu} dx^\mu dx^\nu = -F(r) dt^2 + G(r) dr^2 + r^2(d\theta^2 + \sin^2 \theta d\varphi^2), \quad (39)$$

where we have performed a coordinate transformation to ensure $H(r) = r^2$. It is a straightforward calculation [20] to find the net scattering angle, $\Delta\varphi$, through which an incident null geodesic deviates while passing through such a gravitation field. In terms of the radial coordinate, r_0 , of the point of closest approach of the null geodesic to $r = 0$, the scattering angle is given by:

$$\Delta\varphi + \pi = 2 \left| \int_{r_0}^{\infty} G^{1/2}(r) \left[\left(\frac{r}{r_0} \right)^2 \left(\frac{F(r_0)}{F(r)} \right) - 1 \right]^{-1/2} \frac{dr}{r} \right|. \quad (40)$$

If r_0 is chosen large enough to justify using the asymptotic large- r expansions: $F(r) \simeq 1 - A/r$ and $G(r) \simeq 1 + B/r$, then eq. (40) reduces to

$$\Delta\varphi \simeq \frac{2\xi}{r_0}, \quad (41)$$

where $\xi = \frac{1}{2}(A + B)$. For the nonsingular, Schwarzschild-like solutions ξ is given explicitly by $\xi \simeq \ell + 11\lambda\alpha'/(6\ell) + O(\alpha'^2)$.

After a duality transformation we have $\tilde{F} = 1/F$ and $\tilde{G} = G$. So the scattering of massless string states in the dual metric is again given, in the large- r_0 limit, by eq. (41) but with the replacement

$$\xi \rightarrow \tilde{\xi} \equiv \frac{1}{2}(\tilde{A} + \tilde{B}) = \frac{1}{2}(B - A). \quad (42)$$

For the metric that is dual to the nonsingular, Schwarzschild-like, one, we therefore have $\tilde{\xi} \simeq -2\lambda\alpha'/\ell + O(\alpha'^2)$. Notice that ξ and $\tilde{\xi}$ therefore have opposite signs, and so predict the deflection of an incident massless string state into the opposite directions. Notice also that the relation between ξ with $\tilde{\xi}$ is precisely the same as that obtained earlier between \mathcal{A} and $\tilde{\mathcal{A}}$, or \mathcal{B} and $\tilde{\mathcal{B}}$, *i.e.*: $\ell \rightarrow -2\lambda\alpha'/\ell$, reproducing the result $M \rightarrow -k/(\alpha'M)$. For the self dual metric $(\delta, \gamma) = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ we can see that $A = 0$ and so $\xi = \tilde{\xi}$ as it should, but for

all the other values of δ and γ we find $A \neq 0$ and therefore $\xi \neq \tilde{\xi}$. So in general it appears that the dual backgrounds lead to distinct physical results.

5.2) Equivalent or Not?

Finally then, we attempt to address the question of whether or not string solutions that are related by a duality transformation based on a noncompact symmetry are physically equivalent. Superficially, the calculation of massless-string scattering just described seems to indicate that dual solutions represent physically inequivalent configurations, although as we discuss below, this result is not in itself completely conclusive. We nevertheless believe configurations that are related by dualities based on time translation could be inequivalent, at least in four or more dimensions. We list, in this section, the arguments for, and against, the equivalence of the dual configurations we have considered.

- *1. Two Dimensional Black Holes:* If the dual of the black hole solution really is physically different than the black hole solution itself, then this conclusion runs counter to the conventional wisdom about two-dimensional black holes in string theory.⁵ The two-dimensional solutions arise from a gauged $SL(2, \mathbb{R})/U(1)$ WZW model [4]. There are arguments that in this case target-space duality relates equivalent conformal field theories [15], despite the fact that the masses of the two solutions have opposite signs [5].

The relation between the dual configurations is quite subtle, in the WZW model, since the total space is considered to incorporate several copies of (the maximal analytic extension of) the black hole and the negative-mass naked singularity [5], [7]. Duality then relates different asymptotic regimes of the extended solution. Unfortunately, a similar understanding in the present case is not yet possible, since the conformal field theory representation of the four-dimensional solutions is unknown. As a result there is no guidance as to how negative- and positive-mass spaces might be subsumed within some larger string solution. On a related point, we note that the higher dimensional solutions with a nonvanishing dilaton charge become complex in the region $0 \leq r < \ell$, and so are more pathological in this region than is the usual black hole for $r < 0$. Hence, for a fixed dilaton charge, the positive and negative mass solutions are not naturally linked in a single (real) geometry.

⁵ There is not a flat contradiction, since the above scattering analysis cannot be applied to the two-dimensional examples. See item 4 below, however.

- *2. Momentum and Winding Modes:* There is another argument that can be made against using the calculation of the scattering of massless string states to address the equivalence of dual configurations. This argument is based on the fact that experience with the action of duality on a torus shows that the momentum and winding modes in the symmetry direction should be exchanged by duality. Thus, any physical equivalence between the various black hole solutions would relate the physics of momentum carrying states to that of the winding states in the dual solution.

In all of our four-dimensional solutions, however, the symmetry direction is timelike, as well as being noncompact. Both of these properties make it difficult to introduce the dual ‘winding’ states in a sensible way. Thus, in our comparison of dual solutions, we considered the scattering of momentum-carrying states for both spaces. Within the limitations of our present understanding, this seems to be the only physically relevant comparison that can be made.⁶ Perhaps an analysis along the lines of Ref. [38] would provide some new insight.

The implications of duality for CFT’s in the case of noncompact symmetries has been studied in some detail in Ref. [27]. These authors found that the images under duality of ordinary momentum-carrying modes are a class of *nonlocal* vertex operators (‘vortex lines’). Thus, although this construction can be taken to define an equivalent, dual, conformal field theory in the noncompact case, the resulting theory in general differs from the usual string quantization, as defined as the sum over local string embeddings in the corresponding spacetime. This distinction is probably made clearest by considering the example of standard string propagation in flat Minkowski space. In this case the image spacetime under duality — based on one of the noncompact symmetries, such as translations — is also flat Minkowski space. But since the dual string theory constructed using such a duality transformation includes the nonlocal ‘vortex line’ vertex operators, it is *different* from the starting theory in which no such operators appear.

- *3. Euclidean Solutions:* One might hope to better address these issues by dualizing the euclidean version of our family of dilaton-metric solutions in eq. (22). (One could also consider the general solutions presented in Ref. [13].) In this case, for the (nonsingular) black hole geometry, there is a natural periodic identification of the euclidean time direction — $t \simeq t + \beta$ with $\beta = 2\pi\ell$ — which avoids a conical singularity at $r = \ell$, and so produces

⁶ Further we note that following the example of the torus, one expects the massless states to remain massless under duality

a completely smooth manifold [39]. Of course, such an identification cannot cure the curvature singularities in the solutions having nonzero dilaton charge, and so in this case there is no natural period which suggests itself for the euclidean time direction. It seems likely that the spectrum of the string in the background of a euclidean black hole will be identical to that obtained with the dual solution, if, in the latter, euclidean time is also identified with the inverse period — *i.e.*, $\beta \rightarrow \tilde{\beta} \sim \alpha'/\beta$. This expectation arises because in this case the symmetry direction is compact. Further one expects that the same result holds for dual euclideanized solutions with arbitrary dilaton charge.

This construction does not imply a similar equivalence for the Minkowski signature vacua. The Minkowski-signature problem which corresponds to the euclidean system with time-period β is a system in a heat bath of temperature $T = 1/\beta$. Thus, the duality would relate two Minkowski-signature systems whose temperatures are related by $\tilde{T} \sim 1/(\alpha'T)$. Implicit in our previous discussion is that we are considering the physical equivalence of the zero-temperature vacua for the dual solutions. The reciprocal relationship between the dual temperatures though implies that such a comparison can not be made here, since as $T \rightarrow 0$, the temperature diverges for the equivalent dual heat bath.

Another attempt might be to make a periodic identification of time directly in the original Minkowski-signature solutions. For the static solutions, there is no natural criterion for selecting the time period, but for a solution with a nonvanishing NUT parameter as in eq. (7), a natural period for the time coordinate, $8\pi N$, is suggested by the requirement that the $\theta = 0$ and $\theta = \pi$ axes be free of conical singularities [23].⁷ The resulting solutions would in any case display rather pathological characteristics, similar to those of the Taub-NUT or Misner spaces [23]. The interpretation of winding modes in time-like directions would also remain unclear, and so we did not pursue this approach here.

• *4. Background Gauge Fields:* There is another argument which we bring forward against the idea that the duality transformations in eqs. (15) and (16) always relate physically equivalent solutions. This argument has the advantage that it addresses the potential objections of arguments 1 and 2 above.

Notice that if we consider a general solution which includes background gauge fields,

⁷ If this choice is made, then since $Q_A \leftrightarrow 2N$ under duality, the periodicity of two dual solutions would in general be completely unrelated. They would not, in particular, be related in the usual ‘dual’ way *i.e.* $\tilde{\beta} \sim \alpha'/\beta$.

then the duality transformations become more complicated (see eq. (16) and the surrounding discussion). As was mentioned earlier, our results concerning the action of duality on the mass and the various charges rely crucially on the assumed vanishing of A_t like $1/r$ at infinity. This assumption is crucial because it is required to ensure that the contributions of A_μ to the dual fields — which are typically proportional to A_t^2 — fall off like $1/r^2$, and so do not affect their leading asymptotic behavior.

Before proceeding to our final argument, let us first consider in detail the case where A_t approaches a constant asymptotically,

$$A_t = 2v + \frac{Q_E}{r} + \dots, \quad (43)$$

which leaves the asymptotic behavior of the electric field (10) unchanged. In order to keep the asymptotic form for the field strength, $H_{t\theta\varphi} = -Q_A \sin\theta + \dots$, unchanged the asymptotic behavior of the Kalb-Ramond field must also be modified, because of the Chern-Simons contribution to $H_{\mu\nu\rho}$ (*i.e.* $H = dB - \frac{1}{4}A dA$). The required asymptotic form is:

$$B_{\varphi t} = (Q_A + \frac{v}{2}Q_M) \cos\theta + \dots. \quad (44)$$

After performing the duality transformation, one finds that asymptotically $\tilde{G}_{tt} \rightarrow -1/(1-v^2)^2$. Rescaling the time coordinate, $t = (1-v^2)\tilde{t}$, recovers the desired asymptotic form, $\tilde{G}_{\tilde{t}\tilde{t}} \rightarrow -1$. Notice that with this choice, the asymptotic value of the gauge potential is preserved, *i.e.*, $\tilde{A}_{\tilde{t}} \rightarrow 2v$. It is also necessary to shift the dilaton by a constant in order to recover $e^\phi \rightarrow 1$ as $r \rightarrow \infty$. Now it is straightforward to calculate the new physical parameters which characterize the dual solution, and one finds

$$\begin{aligned} 2\tilde{G}_N\tilde{M} &= \frac{1}{1-v^2} [Q_D - 2v^2G_NM - vQ_E] \\ \tilde{Q}_D &= \frac{1}{1-v^2} [2G_NM - v^2Q_D + vQ_E] \\ 2\tilde{N} &= \frac{1}{1-v^2} [Q_A - 2v^2N] \\ \tilde{Q}_A &= \frac{1}{1-v^2} [(1-2v^2)2N + v^2Q_A] \\ \tilde{Q}_E &= \frac{1}{1-v^2} [(1+v^2)Q_E + 2v(2G_NM - Q_D)] \\ \tilde{Q}_M &= Q_M + \frac{v}{1-v^2} [2N - Q_A] \quad . \end{aligned} \quad (45)$$

In deriving these results, we have assumed that $v^2 < 1$. Notice that these expressions are singular as $v \rightarrow \pm 1$, and further that the limit $v \rightarrow 0$ recovers the previous results of eq. (19). Further, in the first line above, we write \widetilde{G}_N to emphasize that Newton's constant is modified since we made a constant shift of the dilation field so that after the duality transformation it still vanishes asymptotically.

We remark *en passant* that some of the consequences of eqs. (45) can be cast in a form which is similar to the $v = 0$ case:

$$\begin{aligned} 2G_N M + \frac{v}{4} Q_E &\leftrightarrow Q_D - \frac{v}{4} Q_E \\ 2N + \frac{v}{2} Q_M &\leftrightarrow Q_A + \frac{v}{2} Q_M \quad . \end{aligned} \tag{46}$$

Another simple consequence is

$$(2\widetilde{G}_N \widetilde{M} - \widetilde{Q}_D)^2 - \widetilde{Q}_E^2 = (2G_N M - Q_D)^2 - Q_E^2 \quad . \tag{47}$$

Now comes the main point. The result that the physical charges depend on the asymptotic value of A_t , may bear on the physical equivalence of backgrounds related by the duality transformation we have considered. The starting observation is that an arbitrary constant may be added to A_t via a gauge transformation. One could therefore imagine producing a family of gauge-equivalent — and so physically equivalent — solutions to the string equations, all differing only by their asymptotic values for A_t and $B_{\varphi t}$. Under duality, this family of gauge-equivalent solutions yields a family of dual solutions which all have different physical characteristics, as shown in eq. (45), since the asymptotic forms for these fields depend on $2v$, the asymptotic value of A_t . Since this dual family of solutions have different charges, they cannot be physically equivalent.⁸ But this inequivalence is inconsistent with the gauge equivalence of the original family of backgrounds, together with any hypothetical equivalence between each solution and its dual. It would appear that, at best, the original family of gauge-equivalent backgrounds could only be physically equivalent to one of the dual solutions.

⁸ This could be confirmed, for instance, by studying massless-particle propagation in the geometrical optics limit, as in section 5.1. Since this propagation is again given by the null geodesics of the metric (as may be seen by examining the field equations of the low-energy effective theory), it is completely controlled at large distances by the solution's mass, M .

Notice that in the case of a compact symmetry, the same argument cannot be made, since in this case no gauge transformation would be possible to shift A_0 by an arbitrary constant value. (The subscript ‘0’ here is meant to generically indicate the symmetry direction, rather than time.) Instead, backgrounds with a different constant values for A_0 would be considered as physically distinct, since they are characterized by distinct Wilson lines.⁹ Further, let us note that in the noncompact case, the gauge parameter implementing the shift of A_t is divergent as $t \rightarrow \pm\infty$, and does not vanish asymptotically as $r \rightarrow \infty$. Despite these apparently problematic features, gauge potentials A_t related by a constant shift are usually considered physically equivalent.

This argument implies that one of the following three conclusions must be true. Either: (i) Dual field configurations need not be physically equivalent; (ii) Constant shifts of the electrostatic potential, A_t , change the physical content of the theory; or (iii) The transformations described by eqs. (15) and (16) do not correctly describe duality transformations for arbitrary asymptotic values of A_t .

6. Conclusions

The purpose of this paper has been to investigate how duality acts on string black-hole solutions in four (and higher) dimensions. We have obtained the following results:

- (1): We have investigated the action of duality (based on the symmetry of time-translation) on asymptotically flat field configurations. By considering the action of duality on the asymptotic form of a general field configuration in four dimensions, we have been able to show that it simply interchanges the configuration’s mass and dilaton charge, as well as interchanging the axion charge with the Taub-NUT parameter. The electric and magnetic charges remain invariant. We emphasize that this result is completely general, and does *not* hold only for solutions of the leading order string equations. This generality of our result follows since we have based our analysis purely on the asymptotic behaviour of the fields at $r \rightarrow \infty$, and this asymptotic behaviour should be unaffected by higher-derivative string corrections to the equations of motion. We illustrate this general

⁹ A discrete version of the preceding argument might be possible even in this case, based on the large gauge transformations which can shift the Wilson lines by a discrete amount. It would be interesting to investigate the physical equivalence of such theories in more detail.

conclusion in the text, by explicitly constructing the action of duality for a general class of dilaton-metric solutions in four dimensions. In the case of an electrically charged black hole, our result that Q_D and M interchange under duality reproduces some older results of ref. [12]. (The case of higher-dimensions is treated in an Appendix, where similar results are derived.)

It should be borne in mind that our conclusions regarding the exchange of various physical parameters under duality rely on the very plausible assumption that any $O(\alpha')$ corrections to the transformation rules do not affect the values of these charges. Certainly this was found to be the case for the pure dilaton-metric solutions in section 4. The general argument presented there was that dimensional considerations indicate that the $O(\alpha')$ corrections must involve two derivatives of fields, and so they could only modify the asymptotic form of the dual solutions at $O(1/r^3)$. This would not affect the physical parameters of interest here. (A similar result should apply for the analysis in higher dimensions given in the Appendix.)

Of course, the preceding discussion does make the assumption that the gauge potential A_t vanishes asymptotically. If, instead, the potential approaches a constant, *i.e.*, $A_t \rightarrow 2v$, the action of the duality transformations on solutions is still easily described in terms of the transformation of the physical charges. The results in this case, eq. (45), are somewhat more involved though. One may ask whether our results also depend on the particular asymptotic values which we have chosen for the other fields. In fact, they don't. For example, one could have the asymptotic value $G_{tt} \rightarrow -Z^2$. A simple scaling of the time coordinate would reduce this value to -1 , as is used in section 2.2. In principle, we have two options to consider: we can apply the duality transformation after scaling time (yielding the results discussed above), or before. In the latter case, one still finds that a combination of scaling time and shifting the dilaton by a constant yields the same dual background that results from the first choice. Thus this asymptotic value of G_{tt} has no effects on the physics of the dual background. In other fields, the asymptotic values are equally inconsequential for the action of the duality on the physical charges, although the precise implementation may be slightly more involved. For example: via a (time-independent) diffeomorphism, a metric component G_{tr} could be induced. After the duality transformation (15), one would find a new B_{tr} component in the Kalb-Ramond field, but the latter could be removed via a gauge transformation of this field, *i.e.*, $B_{\mu\nu} \rightarrow B_{\mu\nu} + \partial_{[\mu}\omega_{\nu]}$.¹⁰

¹⁰ This is what happens in the case of duality for the Schwarzschild black hole expressed in terms of

The asymptotic value of A_t seems to be distinct in that originally it can be removed via a gauge transformation, but after the duality transformation the presence of this parameter in the other fields is no longer pure gauge, as can be seen by its effect on the physical charges in eq. (45).

- (2): The special case of the four-dimensional ‘Schwarzschild-like’ solutions — *i.e.* those that are nonsingular at the Schwarzschild radius — is particularly interesting. In this case a black hole of mass M is mapped onto a singular configuration whose mass vanishes to the lowest order in α' . By evaluating the next-to-leading corrections in α' to this solution, we are able to obtain the mass, \widetilde{M} , of this dual configuration. We find the intriguing result $\widetilde{M} = -k/(\alpha' M)$, where $k = \lambda\alpha'^2/2G_N^2 = 2\lambda e^{2\phi_0}$. λ here is $\frac{1}{2}$ in the bosonic, and $\frac{1}{4}$ in the heterotic string.

For the superstring, it happens that $\lambda = k = 0$ in the above formula, and so the results of the $O(\alpha'^3)$ corrections to the black-hole configuration are required in order to determine the dual mass. We find, in this case, the negative mass: $\widetilde{M} = -k'/(\alpha'^3 M^5)$, where $k' = 3\zeta(3) e^{6\phi_0}$.

With these results and the analysis given in the Appendix, we have found then that, for all $d \geq 4$, the nonsingular (uncharged) black hole solution gets mapped onto a solution having negative mass, just as for two dimensions [5]. This is *not* a general feature of the dual configurations, as may be seen from the four-dimensional solutions for which both the mass and dilaton charge can be positive.

We would like to remark here that higher derivative terms in the effective string equations do induce α' corrections to the Schwarzschild metric and hence the charges as found in equations (32) and (33). These corrections arise, as discussed in Ref. [14], from the condition of maintaining a regular horizon in the solution to the α' corrected field equations. An analogous result was obtained for the charged dilatonic black hole in Ref. [40], where the condition of maintaining a regular extremal horizon is imposed. This α' dependence of the charges should not be confused as being in contradiction with our statement that duality makes no α' corrections for the corresponding charges. Given a

the Eddington–Finkelstein metric. As was noticed in Ref. [12], the dual metric obtained in this case is identical to what is found using the Schwarzschild metric, but it is supplemented by a nonvanishing $B_{\mu\nu}$. The equivalence of the dualization in Eddington–Finkelstein coordinates is then seen by noticing that the resulting $B_{\mu\nu}$ can be gauged to zero by a transformation $B_{\mu\nu} \rightarrow B_{\mu\nu} + \partial_{[\mu}\omega_{\nu]}$.

solution for which the physical charges have a complicated α' expansion, the action of duality is still to simply trade the mass with the dilaton charge, as well as the axion charge with the Taub-NUT parameter, as in eq. (19).

- (3): We have also addressed the potential inequivalence of the two dual solutions in the case where the symmetry is not compact. Using the calculation of the scattering of massless string states we saw that the dual solutions differed in their predictions. Starting from this we presented several arguments which bear on the equivalence of the dual configurations. In particular our final argument forces us to choose one of the following three options: (i) dual solutions can be physically inequivalent; (ii) constant shifts of the electrostatic potential, A_t , can change the physical content of the theory; or (iii) although the transformation given in eqs. (15) and (16) is a legitimate element of $O(d, d + p)$ [29] (and so generates a new background solution), its identification as the duality transformation rule with background gauge fields is incorrect. We further note that while our final argument of the last section explicitly refers to four spacetime dimensions, it can easily be generalized to other cases.

Setting aside the problems arising with background gauge fields, it appears that Ref. [27] provides the most plausible explanation for the physical equivalence of solutions related by duality based on a noncompact symmetry — namely, this equivalence requires that one of the dual string theories be quantized as vortex gas. In this unconventional string theory, there are no local vertex operators carrying momentum in the symmetry direction, rather one constructs non-local ‘vortex’ operators carrying ‘winding-number’ for this direction. In comparing the dual theories, one must keep in mind this distinction — *e.g.*, in the two-dimensional black hole, the propagation and interactions of tachyons near the singularity are equivalent to the propagation and interactions near the horizon of some dual vortex states, rather than of tachyons. This line of argument suggests that for a given configuration of background fields, there may be more than one consistent string theory. It would be interesting to determine if there are consistent quantization schemes which incorporate both the local and non-local states for noncompact directions, or if this ambiguity only arises in coincidence with background symmetries. In the present context, one unsettling aspect of this approach is that the symmetry direction is the time direction, and so the interpretation of the vortex theory is somewhat perplexing. Certainly much more work is needed in order to clarify these issues.

There are a number of other directions in which the work presented here could be pur-

sued. One would be to examine the transformations given in eqs. (15) and (16) as duality transformation rules with background gauge fields for a compact symmetry. Of particular interest are the effects of large gauge transformations. Another interesting extension of our results may be to examine the implications of duality based on the rotational symmetries of the solutions we have considered. This could be done using either abelian duality, based on constant shifts of the angular coordinate, φ , or non-abelian duality, possibly following Ref. [41], using the entire rotational group.

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Appendix A. Duality in Higher-Dimensions

We record in this appendix the action of duality on the lowest-order static, spherically symmetric, asymptotically flat solutions for spacetime dimensions $d > 4$. For generic values of d , the nontrivial asymptotic fields only include the metric, the dilaton and the gauge potential A_t , and hence the solutions are characterized by their mass, dilaton charge and electric charge. The lowest-order solutions are given explicitly in Ref. [13], and we use these solutions (which are also valid for $d = 4$) as our starting point.

The metric in the Einstein frame may be written as

$$ds^2 = -\frac{U^2}{W^{2(d-3)/(d-2)}} dt^2 + W^{2/(d-2)} V^2 (dr^2 + r^2 d\Omega_{d-2}) \quad (48)$$

where $d\Omega_{d-2}$ is the standard line element on a unit $(d-2)$ -sphere. Recall that for arbitrary dimension d , the sigma model and Einstein metrics are related by $G_{\mu\nu} = e^{-2\phi/(d-2)} g_{\mu\nu}$. The solutions also include the following dilaton and gauge fields

$$\begin{aligned} e^\phi &= W/X \\ A_t &= 2v + \sqrt{x^2 - 1} \left(\frac{1 - U^2 X^{2/(d-2)}}{W} \right) . \end{aligned} \quad (49)$$

These fields are written in terms of the following functions

$$\begin{aligned} W &= \frac{1}{2} \left(1 + x + (1 - x) U^2 X^{2/(d-2)} \right) \\ U^2 &= \left(\frac{\beta}{\alpha} \right)^{2H} \\ V^2 &= (\alpha\beta)^{2/(d-3)} \left(\frac{\alpha}{\beta} \right)^{2H/(d-3)} \\ X &= \left(\frac{\alpha}{\beta} \right)^K , \end{aligned} \quad (50)$$

where

$$\alpha = 1 + \left(\frac{\ell}{4r} \right)^{d-3} \quad \beta = 1 - \left(\frac{\ell}{4r} \right)^{d-3} . \quad (51)$$

The constants H and K must satisfy $H^2 + K^2(d-3)/(d-2)^2 = 1$. These constants as well as x parameterize the three physical charges of these solutions: the mass, the dilaton charge and the electric charge,[13]

$$\begin{aligned}
2G_N M &= \frac{A_{d-2}}{2\pi} \left[H(d-2 + (x-1)(d-3)) - K(x-1)\frac{d-3}{(d-2)} \right] \left(\frac{\ell}{4} \right)^{d-3} \\
Q_D &= \left[2K \left(1 + \frac{x-1}{d-2} \right) - 2H(x-1) \right] \left(\frac{\ell}{4} \right)^{d-3} \\
Q_E &= \sqrt{x^2 - 1} (d-3) \left(4H - \frac{4K}{d-2} \right) \left(\frac{\ell}{4} \right)^{d-3}
\end{aligned} \tag{52}$$

where A_{d-2} is the area of the unit $(d-2)$ -sphere. A fourth constant appears in the gauge potential which is the asymptotic value of the gauge potential, *i.e.*, $A_t \rightarrow 2v$. For $K = 0$, the solutions correspond to charged black holes in which the surface $r = \ell/4$ is a nonsingular event horizon.

Duality transformations map these solutions amongst themselves. We begin by focusing our attention on the dilaton-metric solutions with $A_t = 0$, *i.e.*, $v = 0$ and $x = 1$. Then eq. (52) shows that the mass is proportional to H , and the dilaton charge, to K . Under a duality transformation, one finds $(H, K) \rightarrow (\tilde{H}, \tilde{K})$ where

$$\begin{aligned}
\tilde{H} &= \frac{2(d-3)}{(d-2)^2} K - \frac{d-4}{d-2} H \\
\tilde{K} &= 2H + \frac{d-4}{d-2} K .
\end{aligned} \tag{53}$$

Hence the effect of the duality transformation of the Einstein frame solutions is more complicated in higher dimensions, and one does not find that the duality simply exchanges the mass and the dilaton charge except for $d = 4$. The standard Schwarzschild geometry in isotropic corresponds to $(H, K) = (1, 0)$. This nonsingular black hole is mapped to the dual solution $(\tilde{H}, \tilde{K}) = \left(-\frac{d-4}{d-2}, 2 \right)$. Thus, since $\tilde{H} < 0$, the dual counterpart of the black hole in $d > 4$ dimensions always has a negative mass. It is only for $d = 4$ that the mass vanishes to leading order in α' , and so a more detailed treatment is required to determine its sign. The self-dual solutions in d dimensions are $(H, K) = (\frac{1}{\sqrt{d-2}}, \sqrt{d-2})$, and $\left(-\frac{1}{\sqrt{d-2}}, -\sqrt{d-2} \right)$.

In the general case for $d \geq 4$, one has

$$\begin{aligned}
A_t &\simeq 2v + \frac{Q_E}{(d-3)r^{d-3}} + \dots \\
e^\phi &\simeq 1 - \frac{Q_D}{r^{d-3}} + \dots \\
g_{tt} &\simeq -1 + \frac{2\mu}{(d-2)r^{d-3}} + \dots \\
g_{ij} &\simeq \delta_{ij} \left(1 + \frac{2\mu}{(d-3)(d-2)r^{d-3}} + \dots \right)
\end{aligned} \tag{54}$$

where $\mu = 8\pi G_N M / A_{d-2}$ which reduces to $\mu = 2G_N M$ for $d = 4$. Now it is straightforward to implement a duality transformation. Notice that, as discussed in the main text, when $v \neq 0$ one must scale the time coordinate and shift the dilaton by a constant in order to preserve the desired asymptotic limits for g_{tt} and e^ϕ . One then finds the following physical charges for the dual solutions,

$$\begin{aligned}
\tilde{Q}_E &= \frac{1}{1-v^2} \left[(1+v^2)Q_E + 4v\frac{d-3}{d-2}(\mu - Q_D) \right] \\
\tilde{Q}_D &= \frac{1}{1-v^2} \left[\left(\frac{d-4}{d-2} - v^2 \right) Q_D + \frac{2\mu}{d-2} + \frac{v Q_E}{d-3} \right] \\
\tilde{\mu} &= \frac{1}{1-v^2} \left[2\frac{d-3}{d-2}Q_D - \left(\frac{d-4}{d-2} + v^2 \right) \mu - v Q_E \right]
\end{aligned} \tag{55}$$

where again we have assumed that $v^2 < 1$. Note that for $d = 4$, these results reduce to the corresponding formulae given in eq. (45). Similarly one finds that eq.'s (46) and (47) are replaced by

$$\begin{aligned}
\mu + \frac{v}{2}Q_E &\leftrightarrow 2\frac{d-3}{d-2}Q_D - \frac{d-4}{d-2}\mu - \frac{v}{2}Q_E \\
\left(\frac{2(\tilde{\mu} - \tilde{Q}_D)}{d-2} \right)^2 - \left(\frac{\tilde{Q}_E}{d-3} \right)^2 &= \left(\frac{2(\mu - Q_D)}{d-2} \right)^2 - \left(\frac{Q_E}{d-3} \right)^2
\end{aligned} \tag{56}$$

for $d \geq 4$. Hence the action of the duality transformations on the physical charges is very similar in all dimensions.

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